where $b = a + c$. We have here

$$x^2 = \frac{\rho^2(b-\rho)}{c}, \quad y^2 = \frac{\rho^2(\rho - a)}{c}$$

and the maximum value for x occurs for $\rho = \frac{2}{3}b$. According as $c \gtreqless a/2$, this maximum value really occurs, is coincident with A, or is imaginary. The three cases are illustrated in Fig. 1. That condition (2) is satisfied for the curve in question is easy to verify. Indeed, let ρ_1, ρ_2, ρ_3 be three radii inclined 120° to each other. Then

$$\rho_1 + \rho_2 + \rho_3 = 3a + c\left[\sin^2\theta + \sin^2\left(\theta + \frac{2\pi}{3}\right) + \sin^2\left(\theta - \frac{2\pi}{3}\right)\right] = 3a + \frac{3}{2}c = \frac{3(2a+c)}{2}.$$

The angle φ which the normal makes with the radius vector has for tangent, ρ'/ρ, and $\rho' = 2c\sin\theta\cos\theta$. The curve can be constructed by points in the following manner. Let c be the point where the circle of radius c meets an arbitrary radius. Project C on OY in D, and back on the radius in E. Then $OE = c\sin^2\theta$ and if we carry $EP = a$, the point P is on the curve. If we now draw at O a perpendicular to OP and carry $ON = 2DE = 2c\sin\theta\cos\theta$, then NP is the normal at P.

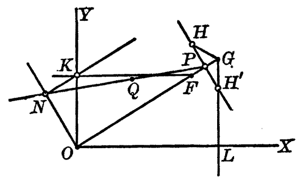


FIG. 2.

The linkage of Fig. 2 enables one to construct the locus of P as well as that of any point Q on its normal by a continuous motion. O, F, and G are fixed pivots. $OF = a$. $FG = GH = GH' = \frac{1}{2}c$. Through H and H' slides the bar HPH' and GH' is kept parallel to OY. The point P, intersection of OG with HH', describes the curve.

In the second part of the linkage, FK is kept parallel to OX, ON is rigidly fixed at right angles to OG and K is a loose pivot, while N slides on ON. If, in the motion of the radius OG, K is describing OY, the bar NP is enveloping the evolute of the curve P and any point Q at a constant distance $PQ = d$ will describe the locus sought in the problem.

503. Proposed by J. W. CLAWSON, Ursinus College, Penn.

If two points A and B invert with respect to a third point O as center of inversion into A' and B', the middle point of the segment AB inverts into the point other than O where the circle of Apollonius (the locus of a point P moving so that $A'P/PB = A'O/OB'$) cuts the circle $OA'B$.

SOLUTION BY HORACE OLSON, Chicago, Illinois.

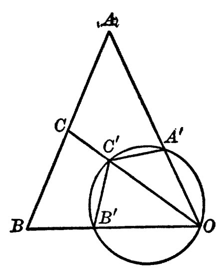
By elementary geometry it is evident that the line AB inverts into the circle passing through O, A', and B'. Let C be the middle point of AB, and C' the point into which it inverts. Draw the lines $A'C'$ and $C'B'$. Triangle ACO is similar to triangle $C'A'O$, and triangle BCO to triangle $C'B'O$. Hence,

$$\frac{A'C'}{AC} = \frac{A'O}{CO}, \quad \text{and} \quad \frac{C'B}{CB} = \frac{B'O}{CO};$$

whence (since $AC = CB$)

$$\frac{A'C'}{C'B'} = \frac{A'O}{B'O},$$

and the proposition is proved.



Also solved by FRANK IRWIN and R. A. JOHNSON.

CALCULUS.

420. Proposed by W. J. GREENSTREET, Stroud, England.

The join of the center of curvature of a curve to the origin is at an angle α to the initial line. Prove that with the usual notation,

$$\frac{d\alpha}{d\psi}\left[\left(\frac{dp}{d\psi}\right)^2 + \left(\frac{d^2p}{d\psi^2}\right)^2\right] = \frac{dp}{d\psi}\cdot\frac{d\rho}{d\psi}.$$